Conformability of a Thin Elastic Membrane Laminated on a Soft Substrate With Slightly Wavy Surface

When laminating a thin elastic membrane on a substrate with surface roughness, three scenarios can happen: fully conformed (FC), i.e., the membrane completely follows the surface morphology; partially conformed (PC), i.e., the membrane remains flat if gravity is not concerned, and partially conformed (PC). Good conformability can enhance effective membrane-to-substrate adhesion strength and can facilitate signal/heat/mass transfer across the interface, which are of great importance to soft electronics laminated on rough bio-tissues. To reveal governing parameters in this problem and to predict conformability, energy minimization is implemented after successfully finding the substrate elastic energy under partially conformable contact.

Four dimensionless governing parameters involving the substrate roughness, membrane thickness, membrane and substrate elastic moduli, and membrane-to-substrate intrinsic work of adhesion have been identified to analytically predict the conformability status and the area of contact. The analytical prediction was found excellent agreement with experimental observations. In summary, an experimentally validated quantitative guideline for the conformability of elastic membrane on soft corrugated substrate has been established in the four-parameter design space. [DOI: 10.1115/1.4032466]

Keywords: thin film, soft substrate, corrugated surface, conformability, adhesion

1 Introduction

Even a highly polished surface has surface roughness. For example, the root-mean-square (rms) roughness of a high-end polished silicon wafer is 0.3 nm [1], and the rms roughness of human skin ranges from 0.03 μm to 45 μm [2]. When a thin membrane is brought into contact with a rough substrate, we expect three contact modes: (1) FC mode as illustrated in Fig. 1(a), i.e., the membrane completely covers the surface of the substrate without any interfacial gap; (2) PC mode as depicted in Fig. 1(b), i.e., some part of the membrane forms intimate contact with the substrate surface while other part of the membrane is suspended; (3) NC mode as shown in Fig. 1(c), i.e., the membrane remains flat if gravity is neglected.

Conformability governs the effective adhesion strength between a thin film and a rough surface. Higher effective adhesion strength can be achieved by improving film-to-substrate conformability. For example, monolayer graphene to silicon adhesion strength is measured to be higher than few layer graphene (FLG) [3], which is attributed to better conformability between monolayer graphene and the silicon substrate [4]. As another example, the feet of geckos and beetles are covered by thin fibers ending with leaflike plates which can be easily bent to well conform to rough contacting surface, which considerably enhances the adhesion strength [5,6]. Moreover, conformability-based metrology has been applied to estimate the adhesion strength between FLG and a precorrugated polydimethylsiloxane [7].

In addition to adhesion strength, conformability of thin membranes on rough surfaces also plays a significant role in the functionality of bio-integrated electronics [8], which have sprung up in recent years due to unlimited potentials in disease monitoring, diagnosis, treatment, as well as human–machine interfaces. Intimate contact between device sheet and bio-tissue is required for effective device–tissue contact. Moreover, effective device–tissue interface mass exchange for sweat monitoring [19,20] and on-demand drug delivery [21] would also benefit from conformable device–tissue contact.
Therefore, a comprehensive mechanistic understanding on the
comformability of thin device sheets on soft bio-tissues can offer
important insights into the design of the mechanical properties of
the bio-integrated devices.

The conformability of a thin membrane on a rigid substrate
with corrugated surface has been studied. For example, in the case
of graphene sheets laminated on silicon substrate, Guo and Huang
developed a theoretical model based on van der Waals interaction
to reveal how the graphene thickness and the surface roughness of
carbon affect the conformability [4]. Wagner and Vella imple-
mented the method of variation of total energy to show that the
substrate profile plays a crucial role in determining the transition
from partial to full conformability [22]. Furthermore, using energy
minimization method, Qiao et al. established a complete theory to
predict the FC, PC, and NC modes of a thin membrane on a rigid
and corrugated substrate [23]. In fact, conformal contact and
effective adhesion strength between a thin elastic plate and a rigid
and randomly rough (e.g., self-affine fractal) substrate have been
studied by Carbone et al. using contact mechanics [24].

When the substrate is a soft solid with surface roughness, it can
deform due to film–substrate interaction and would also try to
conform to the film. Conformability of thin membrane on rough,
 deformable substrate still remains veiled so far due to the
unsure relationship between the membrane and the soft substrate,
especially for PC cases. As a result, the elastic energy stored in
the deformed substrate is difficult to obtain unless the corrugated
substrate surface deforms from one sinusoidal shape to another
sinusoidal shape with the same wavelength but different amplitudes,
which requires FC contact between the membrane and sub-
strate. In this case, analytical solutions of the surface traction and
displacement of the substrate are available [25,26]. Based on those
analytical solutions, substrate elastic energy can be computed as
the work done to the substrate and hence can be used to predict
whether epidermal electronics can fully conform to rough skin
surfaces using the energy minimization method [27].

However, when the film only partially conforms to the sub-
strate, film–substrate interaction and substrate surface displace-
ment are not readily available. We would like to solve for the
partially conformable mode because it is a very common scenario
and we will be able to predict the actual area of contact. In the
case of bio-integrated electronics, bio-tissues like the skin or the
brain generally have a small surface roughness compared with the
wavelength of the corrugation [2,10,29]; therefore, in this paper,
we limit ourselves to the following essential assumptions:

(i) The soft substrate has a slightly wavy surface, i.e., the
amplitude-to-wavelength ratio is smaller than 0.2.
(ii) Within the contact zone, the substrate surface deforms
from one sinusoidal shape to another with the same wave-
length but a different amplitude.
(iii) Shear stress on the membrane–substrate interface is
neglected [25].

Assumption (ii) originates from the FC scenario [27,28] and is
an approximation for slightly wavy surfaces. In Sec. 4, we will
double check if assumption (ii) can lead to the existing FC results.
Since there is no traction applied on the noncontacting substrate
surface, no work is done to the substrate in this area even if there
is displacement. Therefore, there is no need to solve for the dis-
placement of the substrate surface in the noncontacting area
(although we will still provide it in Sec. 4). Since the elastic
energy stored in the substrate equals to the work done to the sub-
strate, as long as we can determine both the displacement and the
traction on the substrate surface within the contact zone, we can
use energy minimization method to analytically search for the
equilibrium configuration and hence predict the conformability
modes as a result of substrate surface roughness, film thickness,
film and substrate moduli, as well as film–substrate intrinsic work
of adhesion. The theoretical model will be discussed in Sec. 2,
and two experimental validations (Ecoflex on skin replica and
polyimide (PI) on brain tissue) are performed in Sec. 3. Further
discussions and concluding remarks are presented in Secs. 4 and
5, respectively.

2 Theoretical Model

A 2D schematic for the PC configuration is given in Fig. 1(d).
For simplicity, the membrane is modeled as a uniform linear elas-
tic membrane with plane strain modulus $E_m$ and thickness $h$.
The soft substrate is assumed to be a precorrugated linear elastic
half space with plane strain modulus $E_s$. Within the Cartesian
coordinate system $x,y$ as defined in Fig. 1(d), the surface profile of
the undeformed substrate is simply characterized by a sinusoidal
equation

$$w_0(x) = h_0 \left(1 + \cos \frac{2\pi x}{\lambda} \right)$$  (1)

where $h_0$ and $\lambda$ denote the semi-amplitude and wavelength of
the undeformed substrate surface, respectively.

When an elastic membrane is laminated on the soft substrate
and starts to conform to the substrate due to interface adhesion,
a contact zone with horizontal projection denoted as $x_c$ is labeled
in Fig. 1(d). Therefore, $x_c = \lambda/2$ represents FC scenario
(Fig. 1(a)), $0 < x_c < \lambda/2$ PC scenario (Fig. 1(b)), and $x_c = 0$
NC scenario (Fig. 1(c)). Due to the membrane–substrate interac-
tion, the soft substrate deforms. Here, we simply postulate that
the surface profile of the substrate within the contact zone
deforms from the initial sinusoidal shape to a new sinusoidal
shape with the same wavelength but a different amplitude, which
can be captured by

$$w_1(x) = h_1 \left(1 + \cos \frac{2\pi x}{\lambda} \right), \quad 0 \leq x \leq x_c$$  (2)

where $h_1$ denotes the deformed semi-amplitude while the wave-
length $\lambda$ remains the same as the initial profile. This assumption
holds all the way till $x_c = \lambda/2$, which means in the FC mode, the
overall substrate surface deforms from one sinusoidal profile to
another with the same wavelength but different amplitudes.

The profile of a PC membrane, $w_2(x)$, as depicted in Fig. 1(d), is
sectional: from $A$ to $B$, i.e., when $0 \leq x \leq x_c$, the membrane
fully conforms to the substrate and thus $w_2(x) = w_1(x)$; from $B$
to $C$, i.e., when $x_c \leq x \leq \lambda/2$, the membrane is suspended, and
$w_2(x)$ is taking a modified hyperbolic shape which will decay to a
parabolic shape when normal strain in the membrane is small, i.e.,
a pure bending condition is assumed [22,23]. Therefore, $w_2(x)$ can
be expressed as

$$w_2(x) = \begin{cases} 
    h_1 \left(1 + \cos \frac{2\pi x}{\lambda} \right), & 0 \leq x \leq x_c \\
    a \left(x - \frac{\lambda}{2}\right)^2 + b, & x_c \leq x \leq \lambda/2 
\end{cases}$$  (3)

where $a$ and $b$ are the two coefficients to be determined by the
continuity condition. Applying the continuity condition at point $B$
where both the profile and the slope of the membrane should be
continuous, i.e., $w_2(x_c) = w_1(x_c)$ and $w_1'(x_c) = w_2'(x_c)$, we can
solve the coefficients $a$ and $b$ to obtain the profile of the
membrane from $B$ to $C$ as

$$w_3(x) = h_1 \left[ \frac{\pi}{\lambda} \sin \frac{2\pi x}{\lambda} \left( x_c - \frac{\lambda}{2} \right)^2 \right]
+ 1 + \cos \frac{2\pi x}{\lambda}, \quad x_c \leq x \leq \lambda/2$$  (4)
To solve for $x_i$ and $h_i$ in Eq. (4), energy minimization method is adopted. The total energy of the system $U_{\text{total}}$ consists of the following four energies:

$$U_{\text{total}} = U_{\text{bending}} + U_{\text{membrane}} + U_{\text{adhesion}} + U_{\text{substrate}}$$

where $U_{\text{bending}}$ is the bending energy of the membrane, $U_{\text{membrane}}$ is the membrane energy associated with tensile strain in the membrane, $U_{\text{adhesion}}$ is the interfacial adhesion energy between the membrane and the substrate, and $U_{\text{substrate}}$ is the elastic energy stored in the substrate, will have to be obtained through contact mechanics analysis. None of the four energies can be neglected in our analysis. Both bending and membrane energies are making significant contributions according to our recent paper of elastic membranes laminated on rigid corrugated substrate [23]. Adhesion energy helps reduce the total energy of the system and is the only negative component out of the four. Nonzero elastic energy stored in the substrate is a deformable object, which is the key to this paper.

The bending energy of the membrane (per unit arc length) is given by

$$U_{\text{bending}} = \frac{2}{\lambda} \left[ \frac{1}{2} \int_A^B E_m h^2 \, ds + \frac{1}{2} \int_B^C E_m h^2 \, ds \right]$$

where $E_m = E_m^2 / 12$ is the plane strain bending stiffness of the membrane, $\kappa$ is its curvature, and $ds$ is the infinitesimal arc length. We use subscript 1 to represent the contact zone, i.e., from $A$ to $B$, and subscript 2 to denote the noncontacting zone, i.e., from $B$ to $C$, as labeled in Fig. 1(d). The membrane energy per unit arc length can be written as

$$U_{\text{membrane}} = \frac{2}{\lambda} \left[ \frac{1}{2} \int_A^B E_m h^2 \, ds + \frac{1}{2} \int_B^C E_m h^2 \, ds \right]$$

For analytical computation of these energies, simplification and nondimensionalization are implemented. Since a slightly wavy surface is considered, the deflection of the membrane is assumed to be small. Therefore, approximations can be applied to simplify the computation of bending energy, which are $\kappa_i \approx w_i^0$ ($i = 1, 2$) and $ds \approx dx$. Hence, the bending energy can be written as

$$U_{\text{bending}} = \frac{2}{\lambda} \left[ \frac{1}{2} \int_0^{\infty} E_m I (w^0_1)^2 \, dx + \frac{1}{2} \int_{\xi_c}^{\infty} E_m I (w^0_2)^2 \, dx \right]$$

$$= \frac{4\pi^2 \beta^2 E_m I}{\lambda^4} D(\xi_c)$$

where

$$D(\xi_c) = \frac{2}{1 - \xi_c} \sin^2(\pi \xi_c) + \pi^2 \xi_c + \frac{\pi}{2} \sin(2\pi \xi_c)$$

and $\xi_c = 2x_c / \lambda$ is the dimensionless parameter that describes the degree of conformability: $\xi_c = 0$ represents NC, $0 < \xi_c < 1$ means PC, and $\xi_c = 1$ denotes FC. If we define three more dimensionless parameters $\beta = 2\pi h_0 / \lambda$, $\eta = i / \lambda$, and $\xi = h_1 / h_0$ and substitute $E_m I = E_m^2 / 12$, we can further express bending energy per unit arc length as

$$U_{\text{bending}} = \frac{4\pi^2 \beta^2 E_m I}{\lambda^4} D(\xi_c) = \frac{E_m^2}{\beta^2 \lambda^2} \eta \xi D(\xi_c)$$

As for the computation of membrane energy and adhesion energy, arc length is taken as $ds \approx \sqrt{1 + (w')^2} \, dx \approx [1 + 1/2(w')^2] \, dx$ (otherwise, strain $\varepsilon(x) = (dx - ds)/dx$ is zero if $ds \approx dx$). Hence, membrane energy becomes

$$U_{\text{membrane}} = \frac{1}{\lambda} \left[ \int_0^{\infty} E_m I (w^0_1)^2 \left( 1 + \frac{1}{2} (w'_1)^2 \right) \, dx + \int_{\xi_c}^{\infty} E_m I (w^0_2)^2 \left( 1 + \frac{1}{2} (w'_2)^2 \right) \, dx \right]$$

$$= \frac{E_m}{\beta^2 \lambda^2} \eta \xi \beta K(\xi_c, \xi \beta)$$

where

$$K(\xi_c, \xi \beta) = \frac{\beta^2}{107520 \pi} \left[ 96(1 + \xi_c)(1 - 25 \beta^2 + 5 \beta^2 \cos(2\pi \xi_c)) \times \sin(\xi \beta)^2 + 35 \left( 144 \pi^2 \xi_c + 60 \beta^2 \pi \xi_c - 3(32 + 15 \beta^2) \right) \times \sin(2\pi \xi_c) + 3(4 + 3 \beta^2) \sin(4 \pi \xi_c) - \beta^2 \sin(6\pi \xi_c) \right]$$

And, adhesion energy can be calculated as

$$U_{\text{adhesion}} \approx -\frac{2\gamma}{\lambda} \int_0^{\infty} \left( 1 + \frac{1}{2} (w'_1)^2 \right) \, dx = -\gamma E(\xi_c, \xi \beta)$$

where

$$E(\xi_c, \xi \beta) = \frac{\left( \frac{1}{\xi_c} \left( 1 + \frac{1}{4} \beta^2 \xi^2 \right) - \frac{(\xi \beta)^2}{8\pi} \sin(2\pi \xi_c) \right)}{4\pi}$$

The calculation of the elastic energy stored in the substrate $U_{\text{substrate}}$ is not as straightforward because the traction between membrane and substrate is not readily known. According to assumption (ii), displacement of the substrate surface within the contact zone can be calculated as

$$u(x) = w_1(x) - w_0(x) = \left( h_1 - h_0 \right) \left( 1 + \cos \left( \frac{2\pi x}{\lambda} \right) \right), \quad 0 \leq x \leq x_c$$

For the surface traction $P(x)$ over the contact zone as labeled in Fig. 2, if we just focus on the elastic substrate with a slightly wavy surface, Johnson [30] has a conclusion that is directly applicable to our situation. He claimed that $P(x)$ can be comprehended by the superposition of a compressive pressure $P_1(x)$ and a tensile pressure $P_2(x)$, which follows as:

$$P(x) = P_1(x) + P_2(x), \quad 0 \leq x \leq x_c$$

Here, $P_1(x)$ is the so-called “bearing pressure” which induces a sinusoidal surface displacement $u(x)$ on a soft substrate with either flat or a slightly wavy surface. The contact of two slightly wavy half-planes in the absence of adhesion (Fig. 3a) was first
Zone where sinusoidal displacement is induced in the contact pressive pressure. The profile of correugation, following Westergaard’s solution [31], the bearing pressure distribution over contact zone, i.e., $0 < x < x_c$.

When subjected to uniform external pressure, a rigid body with a slightly wavy surface is compressed against an infinitely large elastic sub-

Solving Eq. (20) yields

\[ U_{\text{substrate}} = 2 \int_0^c \frac{1}{2} u(x)P(x)dx = \frac{1}{2} \int_0^c u(x)[P_1(x) + P_2(x)]dx
\]

\[ = \frac{(h_0 - h_1)^2 E_s \pi}{k}[F_1(\hat{x}) - F_2(\hat{x})]
\]

\[ = \frac{E_s \beta^2 (1 - \zeta^2)^2}{4\pi}[F_1(\hat{x}) - F_2(\hat{x})]
\]
where $F_1(x_c)$ and $F_2(x_c)$ are the two dimensionless functions

$$F_1(x_c) = \frac{1}{2} \int_0^\infty \left( 1 + \cos \frac{2\pi x}{\lambda} \right) \cos \frac{\pi x}{\lambda} \times \sqrt{\left( \sin \frac{\pi x}{\lambda} \right)^2 - \left( \sin \frac{\pi x}{\lambda} \right)^2} \, dx$$

$$F_2(x_c) = \frac{1}{2} \int_0^\infty \left( 1 + \cos \frac{2\pi x}{\lambda} \right) \left( \sin \frac{\pi x}{\lambda} \right)^2 \times \cos \frac{\pi x}{\lambda} \left[ \frac{\left( \cos \frac{\pi x}{\lambda} \right)^2 - \left( \cos \frac{\pi x}{\lambda} \right)^2}{4} \right] \, dx$$

Taylor expansion of $P_1(x)$ and $P_2(x)$ up to $O(x^9)$ is applied to numerically solve $U_{\text{substrate}}$.

Hence, the total energy of the system can be explicitly expressed as

$$U_{\text{total}} = \frac{E_m}{2} \frac{\beta^2}{12} \eta^3 D(x_c) + E_m \eta \left( \beta \xi^4 K(x_c, \xi \beta) - \gamma E(x_c, \xi \beta) \right) + E_m \beta (1 - \xi^2)^2 \frac{4\pi}{\mu} \left( F_1(x_c) - F_2(x_c) \right)$$

Through dimensional analysis, we want to introduce two additional dimensionless parameters $x = E_m/E_n$ and $\mu = \gamma/(E/l)$, which are membrane-to-substrate modulus ratio and normalized interface intrinsic work of adhesion. Finally, the normalized total energy becomes

$$\hat{U} = U_{\text{total}} \frac{E_n}{E_m \beta^2} = \frac{\xi^2}{12} \eta^3 D(x_c) + x \eta \xi^4 \beta^2 K(x_c, \xi \beta) - \frac{\mu}{\beta^2} E(x_c, \xi \beta) + \frac{(1 - \xi^2)^2}{4\pi} \left( F_1(x_c) - F_2(x_c) \right)$$

which is a function of four dimensionless input parameters: $\beta = 2\pi h_0/\lambda$, $\gamma = \gamma/E_n$, $x = E_m/E_n$, and $\mu = \gamma/(E/l)$, which are physically interpreted as normalized roughness of corrugated substrate ($\beta$), normalized membrane thickness ($\eta$), membrane-to-substrate modulus ratio ($x$), and normalized membrane-substrate intrinsic work of adhesion ($\mu$), respectively. In addition, there are two unknown dimensionless parameters: $x_c = 2x_c/\lambda$ and $\xi = h_1/h_0$, which once solved can yield the contact zone and the amplitude of the deformed substrate. By fixing $\beta, x, \mu$, and $\eta$, minimization of Eq. (25) with respect to $x_c$ and $\xi$ within the domain confined by $0 \leq x_c \leq 1$ and $0 \leq \xi \leq 1$ will give us the equilibrium solution, which can be visualized as the global minimum of the 3D plot of the normalized total energy landscape as a function of $x_c$ and $\xi$.

3 Experimental Validation

With the total energy obtained in Eq. (25), we are now ready to implement the energy minimization method to predict conformability conditions of thin membranes laminated on soft, corrugated substrates. Two experiments in the literature are adopted to validate our model.

3.1 Ecoflex Membrane on Skinlike Substrate.

Epidermal electronics can be exploited for many clinical and research purposes. Due to the ultimate thinness and softness of epidermal sensors, laminating them on microscopically rough skin surface leads to fully conformal contact, which can maximize the signal-to-noise ratio while minimize motion artifacts, as evidenced in Ref. [34]. To optimize the design of epidermal electronics for human–machine interface, Jeong et al. [13] tested the conformability of elastomer membranes (Ecoflex, Smooth-On, USA) of various thicknesses on an Ecoflex replica of the surface of human skin. Membrane–substrate conformability is clearly revealed by the cross-sectional scanning electron microscopy images (Fig. 2(a) in Ref. [13]): 5 µm thick membrane can achieve full conformability to the substrate, 36 µm thick membrane only PFC to the substrate, whereas membranes with thickness of 100 µm and 500 µm remained NC at all. Basic parameters that can be extracted from the experiments are: substrate roughness $h_0 = 50$ µm, $\lambda = 250$ µm, plane strain moduli of membrane, and substrate $E_s = E_m = 92$ kPa [13]. Since the conformability experiments were carried by placing Ecoflex membrane on Ecoflex-based skin replica, we assume the interface intrinsic work of adhesion to be $\gamma = 50$ mJ/m² according to our recent experimental measurements on the work of adhesion between different types of elastomers [35]. Based on those given parameters, the four dimensionless parameters are computed as follows: $\beta = 1.2, x = 1, \mu = 0.003,$ and $\eta = 0.02, 0.144, 0.4,$ and 2, which corresponds to the four different experimental thicknesses of the membrane $t = 5$ µm, 36 µm, 100 µm, 500 µm, respectively. Normalized total energy given by Eq. (25) of each $\eta$ is calculated, and the energy landscape $\hat{U}$ versus $x_c$ and $\xi$ is plotted in Figs. 5(a)–5(d). When $\eta = 0.02$ (Fig. 5(a)), the global minimum falls at $x_c = 1$ and $\xi = 0.88$, as highlighted by the red dot in the figure and the inset. $x_c = 1$ indicates full conformability and $\xi = 0.88$ suggests that the substrate is flattened to a new amplitude of $h_1 = 0.88h_0$. When $\eta = 0.144$ (Fig. 5(b)), the minimal energy locates at $x_c = 0.9, \xi = 0.65$, indicating a PC scenario where contact zone only covers about 9% of the wavelength. As for $\eta = 0.4$ (Fig. 5(c)) and $\eta = 2$ (Fig. 5(d)), the minimal energy points are both at $x_c = 0$, $\xi = 1$, suggesting that the membrane is nonconformal to the substrate and the substrate is not deformed at all. Therefore, our predictions of conformability for four different membrane thicknesses are in excellent agreement with the experimental findings.

By fixing the substrate morphology $\beta = 1.2$ (i.e., $h_0 = 50$ µm and $\lambda = 250$ µm), Fig. 6 predicts the conformability as a function of the other three parameters $x, \mu$, and $\eta$. By numerically solving the minimization problem above, a 3D plot in Fig. 6(a) shows two critical surfaces dividing FC/PC and PC/NC. It is obvious that the FC condition can be achieved at small $\eta$, i.e., thinner membrane, small $x$, i.e., softer membrane compared to the substrate, and large $\mu$, i.e., strong membrane–substrate intrinsic work of adhesion. On the contrary, NC condition most likely occurs at large $x$, large $\eta$, and small $\mu$.

To better illustrate the effect of individual variables, we choose to fix three variables and only change one at a time. For example, in Fig. 6(b), $\hat{x}$ is plotted as a function of $\eta$ in the top axis and $t$ in the bottom axis with $\beta = 1.2, x = 1$, and $\mu = 0.003$ fixed. It is evident that as the film thickness grows from 0, the conformability goes from FC to PC and finally NC. While the transition from PC to NC is smooth, the transition from FC to PC is abrupt, which suggests a significant drop (>77%) of contact area from FC to PC. Similar jump has been observed for FLG conforming to silicon substrate [36] and elastic membrane laminated on rigid, corrugated substrate [23]. More analysis on how different substrate morphologies affect snap-through transition can be found in Ref. [22]. Quantitatively, full conformability requires $\eta < 0.03$, i.e., $t > 7.5$ µm. When $\eta > 0.28$, i.e., $t > 70$ µm, there is no conformability at all. When $0.03 < \eta < 0.28$, i.e., when $7.5$ µm $< t < 70$ µm, the contact area of the PC scenario can be determined. The three black dots indicate the three different membrane thicknesses tested in the experiments by Jeong et al. [13], which are fully consistent with our prediction.

Since the original epidermal electronics was fabricated on 30 µm thick Ecoflex [12], the conformability of a 30 µm thick membrane on an Ecoflex skin replica substrate has been predicted. In order to show the effect of adhesion energy and membrane modulus over wide range, $x_c$ versus $\mu$ (or $\gamma$) and $x_c$ versus $\eta$
Fig. 5 Normalized total energy landscape of Ecoflex membrane of four different thicknesses (four different $g$'s) laminating on Ecoflex skin replica, where $x = 1$, $\beta = 1.2$, and $\mu = 0.003$. Global minima are labeled by red dots. (a) When $\eta = 0.02$, $x_c = 1$, and $\xi = 0.88$, it indicates FC. (b) When $\eta = 0.0144$, $x_c = 0.09$, $\xi = 0.65$, it predicts PC. (c) When $\eta = 0.4$ and (d) when $\eta = 2$, $x_c = 0$, and $\xi = 1$, it suggests NC.

Fig. 6 (a) Surfaces dividing FC/PC and PC/NC when $b = 1.2$ (i.e., $h_0 = 50 \mu m$ and $\lambda = 250 \mu m$) is fixed. (b) Contact area $x_c$ versus $\eta$ on the top or $t$ in the bottom when $\beta = 1.2$, $x = 1$, and $\mu = 0.003$. (c) Contact area $x_c$ versus $\mu$ when $\beta = 1.2$, $x = 1$, and $\eta = 0.12$. (d) Contact area $x_c$ versus $x$ when $\beta = 1.2$, $\mu = 0.003$, and $\eta = 0.12$. 
of the brain gyrus is determined to be \( l \). According to the experimental pictures [10], roughness brain, while the 76 \( m \) thickness electrodes achieved full conformability to the feline brain in vivo. It turned out that the 2.5 \( m \) gold layer is only 150 \( n \)m thick, which is much thinner than the thinnest PI they used (2.5 \( m \)). The conformability of electrodes with two different PI thicknesses (2.5 \( m \) and 76 \( m \)) was tested on a feline brain in vivo. In summary, Fig. 6 offers a quantitative guideline towards conformable skin-mounted electronics in the four-parameter design space.

### 3.2 PI Membrane on a Feline Brain In Vivo

In addition to human skin, brain is another soft organ with surface roughness that can prevent intracranial electrodes from conformal contact with the cortex. To retrieve electrocorticography with high amplitude to \( h = 0.24 \) \( m \) and \( \lambda = 11.86 \) \( m \), which yields \( \beta = 0.13 \). By neglecting the gold layer, the modulus of the electrodes is given by PI modulus: \( E_m = 2.8 \) \( GPa \) [10]. The modulus of the brain is found in literature as \( E_\gamma = 50 \) \( kPa \) [37]. Hence, the membrane–substrate modulus ratio is computed as \( \alpha = 0.06 \), 0.002, 0.0002 are fixed. We use three thicknesses (2.5 \( m \), 12 \( m \), and 76 \( m \)) (Fig. 7(a)) - (c), global minimal energy falls at \( \tilde{x} = 1 \) and \( \tilde{c} = 0.9 \) as labeled by the red dot. It means the electrode is predicted to fully conform to the brain while the brain was slightly flattened by reducing the amplitude to \( h_1 = 0.9h_0 \).

When \( \eta = 0.001 \) (i.e., \( t = 12 \) \( m \)) (Fig. 7(b)), the global minimal minimum locates at \( \hat{x} = 0.12 \) and \( \hat{c} = 0.86 \), as highlighted by the red dot in the figure, which indicates a PC scenario. When \( \eta = 0.006 \) (i.e., \( t = 76 \) \( m \)) (Fig. 7(c)), the minimal energy occurs at \( \hat{x} = 0 \) and \( \hat{c} = 1 \), suggesting that the membrane is not able to conform to the cortex at all. To offer a holistic picture of the effect of electrode thickness on conformability, Fig. 7(d) plots \( \hat{x} \) as a function of \( \eta \) as the top axis and \( t \) as the bottom axis when \( \beta = 0.14 \), \( \gamma = 56,000 \), and \( \mu = 2.4 \times 10^{-4} \).
modes again suggests that there is an upper limit in the maximum contact area (23% of the total surface area) under the PC condition and hence FC mode is strongly preferred for effective measurements and treatments.

4 Discussion

4.1 Double-Checking Assumption (ii) Under FC Condition. Assumption (ii) dictates that the substrate surface deforms from one sinusoidal shape to another sinusoidal shape over the contact zone. This is inspired by the FC scenario in which the substrate surface undergoes a sinusoidal deformation over the entire wavelength when the membrane is fully attached on it, in which case the traction exerted on the substrate is also sinusoidal over the contact zone. This is inspired by the FC scenario in which the membrane is much smaller than the wavelength of the corrugated substrate, i.e., \( \eta = t/\lambda \ll 1 \). However, when the thickness of the film is comparable or even larger than the wavelength of the substrate, this assumption no longer holds, which is referred as a thick slab. When a thick but soft slab is laminated on a corrugated substrate, the lower surface of the slab will deform to fill the cavity between the substrate while the upper surface of the slab will stay almost flat. As a result, the slab needs to be modeled as an elastic body instead of a beam (or plate). Hence, the total energy given by Eq. (25) is no longer reliable when, for example, \( \eta = 2 \) (i.e., \( t = 500 \mu m \)) in Fig. 6. The contact problems of a thin elastic plate and an elastic body making contact with a randomly rough hard surface were studied by Persson and coworkers [24], in which the elastic energy needed to deform a large thin plate \( U_{plate} \) and to deform a semi-infinite elastic solid \( U_{solid} \) so that they make full contact with a substrate cavity of diameter \( \lambda \) and height \( h \) are given as

\[
U_{plate} \sim E\lambda^2 \left( \frac{h}{\lambda} \right)^2
\]  
(33)

\[
U_{solid} \sim E\lambda^3 \left( \frac{h}{\lambda} \right)^2
\]  
(34)

respectively, where \( E \) is the Young’s modulus of the plate or solid on the top. If \( t \ll \lambda \), the elastic energy stored in plate is much smaller than the elastic energy stored in a thick solid. Therefore, the thin plate is elastically softer than a thick slab and hence easier to conform to the substrate.

5 Conclusions

Using the method of energy minimization, this paper develops an analytical model to determine the conformability of a thin elastic membrane placed on a soft substrate with a slightly wavy surface. Four dimensionless governing parameters have being identified. Although the effect of each parameter is monotonic, abrupt transition from FC to PC has been observed for all the parameters. Analytical predictions of the conformability of Ecoflex membrane on Ecoflex-based skin replica and PI membrane on an in vivo feline brain have found excellent agreement with the experimental observations of conformability. Furthermore, critical membrane thickness, membrane–substrate intrinsic work of adhesion, and membrane to substrate stiffness ratio are identified for full conformability. This model hence provides a viable method to predict the conformability and contact area between thin elastic membrane and soft substrate with slightly wavy surface. It also offers a guideline for the design of the electronic membrane as well as the bio-electronic interface to achieve high conformability.

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